Analytical theory of zone plate efficiency

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The theory is developed which enables us to derive explicit formulas for the efficiency of thick zone plates. The theory employs the method of coupled wave equations. The solutions obtained can be applied to the large class of zone plates and diffraction structures which are locally equivalent to sliced multilayers. Based on this theory we find limits to the diffraction efficiency of zone plates due to refraction and absorption indexes of materials. The results can be applied in x-ray, neutron, and atom optics.

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I. INTRODUCTION

Recent advances and prospects for high resolution imaging with x-ray [1], neutron [2], and atom beams [3] are to a large extent associated with zone plates [4]. They are systems of alternating annular layers of two materials (one of which can be the vacuum) with different optical constants. Every pair of neighboring layers corresponds to a Fresnel zone, the total number of which reaches 500, and the width of the outermost zone can be of the order of the wavelength λ of the radiation used for imaging.

From the point of view of the wave theory a zone plate is a typical multiscale object. It has at least three characteristic lengths: the width of the zone (angstrom scale), zone plate thickness (submicrometer or micrometer), and focal length (submillimeter or millimeter). This multiscale character presents severe difficulties for analytical and numerical description of zone plate properties.

High throughput zone plates are extremely desirable for current microscopy experiment as the intensity of x-ray, neutron, and atom beams is often not enough to achieve reasonable exposure time and spatial resolution.

The estimations of diffraction efficiency and design of zone plates are usually based on Fresnel-Kirchhoff theory, which predicts 10% efficiency for the amplitude zone plate and 40% efficiency for the phase zone plate [5,6]. Some possibilities were indicated to increase these values by varying the density of materials inside the zones [7].

However, in the conventional Fresnel-Kirchhoff theory the zone plate is considered as a screen and diffraction effects inside the zone plate body are neglected. This is justified if the zone plate thickness $x < \delta^2/\lambda$, where δ is the width of the outermost zone. But this condition is violated for thick zone plates which are necessary to use to obtain high diffraction efficiency. So more rigorous calculations coming beyond the framework of the standard Fresnel-Kirchhoff theory are needed. Maser and Schmahl [8] demonstrated a numerical approach to the problem using coupled wave equations for diffraction amplitudes and showed that local diffraction efficiency of x-ray zone plates considerably increases if the proper thickness and slant of the zones are provided.

In this paper we develop an analytical approach to calculation of diffraction efficiency of zone plates. We use the analogy between zone plates and sliced multilayers, the theory of which was considered in [9]. In Sec. II the general approach and basic equations are given. In Sec. III coupled equations for diffraction amplitudes are solved near the Bragg resonance, which is the most interesting case for various applications. In Sec. IV the final expression for diffraction efficiency is obtained in terms of zone plate aperture, thickness, optical constants of materials, and their spacing ratio (i.e., the ratio of widths of material layers inside one zone). Further consideration is based on this result. In Sec. IV A it is shown that to obtain the zone plate with highest diffraction efficiency the value of the spacing ratio should be chosen exactly in the same way as it is done in the theory of multilayer mirrors [10]. Then in Sec. IV B and IV C the dependence of efficiency on the thickness is considered for zone plates of various types. New values of upper and lower limits of diffraction efficiency follow from this consideration. In Sec. V the theory is applied to pattern and sputter-sliced zone plates made of Ni and Be. These materials are used now in imaging experiments with x-ray lasers and synchrotron radiation [5,11]. As practical examples in this paper we take only x-ray zone plates. However, the same approach can be used for analyses of Fresnel zone plates used in neutron and atom optics.

II. BASIC EQUATIONS

A. Foreword

For the sake of completeness we begin with the known formulas of the zone plate theory [5] and derive them considering the zone plate as the combination of local gratings.

Let us consider a zone plate that images at a distance z' a point source located at a distance z from the zone plate (see Fig. 1). Assume that the image is produced

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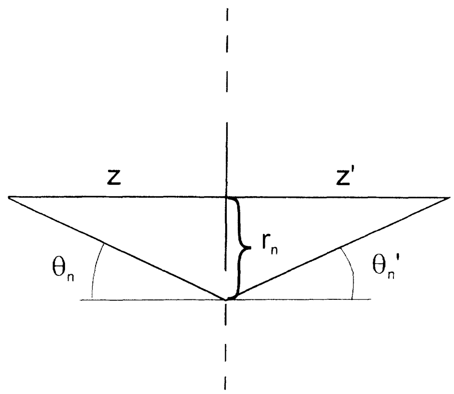

FIG. 1. Ray diagram for imaging zone plate.

by narrow beams and each beam emitted by the source at an angle θ interacts with the zone plate as with the infinite grating having the local period

$$d_n = r_n - r_{n-1}, \qquad (1)$$

where r_n is the radius of the nth zone. So we define a zone as one period of local grating.

To produce the image every small part of the zone plate (i.e., every local grating) should direct the first diffraction order of the incident beam toward the point z'. This is provided if the grating equation

$$d_n(\sin\theta_n + \sin\theta_n') = \lambda, \qquad (2)$$

where

$$\sin\theta_n = \frac{r_n}{\sqrt{r_n^2 + z^2}}, \qquad \sin\theta_n' = \frac{r_n}{\sqrt{r_n^2 + z'^2}} \qquad (3)$$

is valid for any θ_n. Then taking into account (1) we obtain

$$(r_{n+1} - r_n)\left(\frac{r_n}{\sqrt{r_n^2 + z^2}} + \frac{r_n}{\sqrt{r_n^2 + z'^2}}\right) = \lambda\ . \qquad (4)$$

This is the recurrent formula for zone radius r_n. Strictly speaking it is true since the variation of the period is small as compared with the period:

$$d_{n+1} - d_n \ll d_n .$$

That is why it is reasonable to consider the zone number n in (4) as a continuous parameter and replace (4) by an equivalent differential equation:

$$\frac{dr_n}{dn}\left(\frac{r_n}{\sqrt{r_n^2 + z^2}} + \frac{r_n}{\sqrt{r_n^2 + z'^2}}\right) = \lambda, \qquad (5)$$

the solution of which can be written as

$$\sqrt{r_n^2 + z^2} + \sqrt{r_n^2 + z'^2} = \lambda n + \sqrt{r_0^2 + z^2} + \sqrt{r_0^2 + z'^2}, \qquad (6)$$

where r_0 is the radius of the 0th zone, or in other words, r_0 is the radius of the central obscured zone.

Formula (6) is the equation for the zone radius r_n. It is exactly the same as one usually obtained by calculation of the phase differences for the rays deflected by neighboring zones [5]. Note that the local period d_n in (1) and (2) is the sum of the widths of transparent and opaque zones. For arbitrary diffraction order m the local period can be written in the form similar to (2):

$$d(\theta) = \frac{m\lambda}{\sin\theta + \sin\theta_m}, \qquad \tan\theta_m = \frac{z}{z'}\tan\theta. \qquad (7)$$

Let us now apply the local grating approach to calculation of zone plate efficiency.

B. Total diffraction efficiency

In order to find the amplitudes of diffracted waves we must solve the wave equation for the plane waves incident onto the local grating:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k^2 E + k^2\epsilon(x,y)E = 0, \quad k = \frac{2\pi}{\lambda} \qquad (8)$$

where $\epsilon(x,y)$ is the difference between dielectric constant and 1. It is a periodic function of y which vanishes outside the zone plate $\epsilon(x,y) = \epsilon(x, y+d)$ (Figs. 2 and 3).

The exact solution of (8) can be written in the form (see, for example, [12])

$$E(x,y) = \exp(iky\sin\theta)f(x,y) = \sum_{m=-\infty}^{+\infty} f_m(x)\exp\left[iy\left(k\sin\theta - \frac{2\pi m}{d}\right)\right], \qquad (9)$$

where θ is the angle of incidence of the plane wave onto the grating, $f(x,y)$ is a periodic function of y, $f_m(x)$ is the amplitude of the mth diffraction order.

Let I and $\vec{n}$ characterize the local intensity and the direction of propagation of the beam incident onto the zone plate (or in the general case, arbitrary dispersive element); $I_m = I \mid f_m \mid^2$ and $\vec{n}_m$ are the intensity and direction of propagation of the mth diffraction order (see

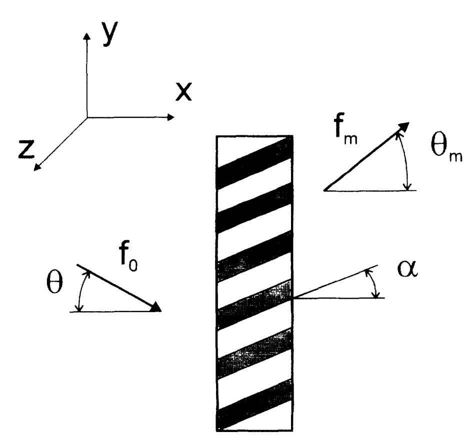

FIG. 2. A local grating of the zone plate with slanted zones.

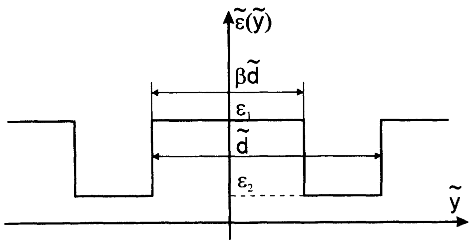

FIG. 3. Shape function of local grating permittivity of the zone plate produced by lithographic or sputter-sliced methods.

also Fig. 1). Then the total diffraction efficiency of the mth order (TDE) is given by the ratio of total amounts of diffracted and incident energies:

$$E_{\text{TDE}} = \frac{\int I_m(\theta)\vec{n}_m \cdot d\vec{\sigma}}{\int I(\theta)\vec{n} \cdot d\vec{\sigma}} = \frac{\int I(\theta) \mid f_m \mid^2 \cos\theta_m d\sigma}{\int I(\theta)\cos\theta d\sigma}, \tag{10}$$

where according to (7) $\sin\theta_m = \lambda m/d - \sin\theta$. The integrals in (10) are taken over the entire surface of the zone plate (dispersive element).

TDE depends on the zone plate geometry and also on the structure and position of the source. Let us calculate TDE for the point source located at the axis of the zone plate (Fig. 1). Then we have $I = I_0\cos^2\theta/4\pi z^2$ and $d\sigma = 2\pi z^2 \sin\theta/\cos^3\theta d\theta$, where I_0 is the total power radiated by the source. Substituting this into (10) we obtain

$$E_{\text{TDE}} = C^{-1}\int_{\theta_1}^{\theta_2} \mid f_m(\theta) \mid^2 \frac{\cos\theta_m}{\cos\theta}\sin\theta d\theta, \tag{11}$$

where $C = \cos\theta_1 - \cos\theta_2$, and θ_1 and θ_2 are the smallest and largest angles of zone plate aperture. Thus TDE of the zone plate in the mth order is given by formula (11) in terms of local diffraction amplitude $f_m(\theta)$.

C. Coupled equations

Diffraction amplitudes $f_m(\theta)$ obey the system of coupled equations which can be easily obtained by substituting of expansion (9) into wave equation (8):

$$f_m''(x) + k^2\kappa_m^2 f_m(x) + k^2 \sum_{m'\neq m} f_{m'}(x)\epsilon_{m-m'}(x) = 0, \tag{12}$$

where

$$\kappa_m^2 = 1 - \left(\sin\theta - \frac{\lambda m}{d}\right)^2 + \epsilon_0(x) = \cos^2\theta_m + \epsilon_0(x), \tag{13}$$

and

$$\epsilon_m(x) = \frac{1}{d}\int_0^d \epsilon(x,y)\exp\left(i2\pi m\frac{y}{d}\right)dy. \tag{14}$$

Till now we used the only suggestion that the zone plate is locally (at any given θ or r) characterized by permittivity, which is a periodic function of y. Let us consider the zone plate with slanted zones (see Fig. 2). Then the local grating becomes a full analog of sliced multilayer. As it was shown in [9] in this case the truncated system of coupled equations corresponding to (12) has exact analytical solution. We will apply a similar approach to calculation of zone plate TDE [see (11)].

So the zone plate with slanted zones is locally considered as a sliced multilayer. This means that $\epsilon(x,y) = \epsilon(-x\sin\alpha + y\cos\alpha)$. Then coupling coefficients $\epsilon_m(x)$ take the exponential form

$$\epsilon_m(x) = \exp\left(i2\pi m\frac{x}{d}\sin\alpha\right)\epsilon_m, \tag{15}$$

$$\epsilon_m = \frac{1}{\tilde{d}}\int_0^{\tilde{d}} \tilde{\epsilon}(\tilde{y})\exp\left(i2\pi m\frac{\tilde{y}}{\tilde{d}}\right)d\tilde{y}, \tag{16}$$

where $\alpha(\theta)$ is the local angle of zone inclination, and $\tilde{d}$ and $\tilde{\epsilon}(\tilde{y})$ are the period and the shape function of permittivity of the local grating.

The present technology (electron-beam lithography or sputter-sliced technique [5]) produces the zone plate as a set of alternating well separated zones of two materials 1 and 2. For this case the shape function is shown in Fig. 3 and coefficients ϵ_m can be written as

$$\begin{aligned} \epsilon_m &= \beta\epsilon_1 + (1-\beta)\epsilon_2, \qquad m = 0, \\ \epsilon_m &= (\epsilon_1 - \epsilon_2)\frac{\sin\pi\beta m}{\pi m}, \qquad m \neq 0. \end{aligned} \tag{17}$$

The zone plate efficiency (10) is expected to be large if the resonance effect takes place in all the local gratings. This means that incident radiation is mainly transformed into one of diffraction orders and the intensity of other orders is very small and in the system of Eq. (12) they can be neglected. Then we obtain the system of two coupled equations:

$$\begin{aligned} &f_0''(x) + k^2\kappa_0^2 f_0(x) + k^2 f_m(x) \times \exp\left(-i2\pi m\frac{x}{d}\sin\alpha\right)\epsilon_m = 0, \\ &f_m''(x) + k^2\kappa_m^2 f_m(x) + k^2 f_0(x) \times \exp\left(i2\pi m\frac{x}{d}\sin\alpha\right)\epsilon_m = 0, \end{aligned} \tag{18}$$

where ϵ_m are given by (17) and κ_0,κ_m by (13).

III. SOLUTION NEAR THE BRAGG RESONANCE

Exact analytical solution of (18) was used in [9] for the theory of sliced multilayers. This solution can be considerably simplified if we introduce slowly varying amplitudes $\phi_0(x)$ and $\phi_m(x)$:

$$f_0(x) = \phi_0(x)\exp(ik\kappa_0 x), \quad f_m(x) = \phi_m(x)\exp(ik\kappa_m x). \tag{19}$$

Substituting (19) into (18) and neglecting the second derivatives of ϕ_0 and ϕ_m we obtain

$$i\phi_0' + \frac{k\epsilon_m}{2\kappa_0}\phi_m \exp(iBkx) = 0\ , \quad i\phi_m' + \frac{k\epsilon_m}{2\kappa_m}\phi_m \exp(-iBkx) = 0, \tag{20}$$

where

$$B = (\kappa_m - \kappa_0) - \frac{\lambda m}{d}\sin\alpha. \tag{21}$$

The diffraction amplitude in the mth order is found by solving Eq. (20) with initial conditions $\phi_0(0) = 1, \phi_m(0) = 0$:

$$\phi_m(x) = \frac{i\epsilon_m}{\kappa_m\Delta}\exp\left(-ikx\frac{B}{2}\right)\sin\left(kx\frac{\Delta}{2}\right), \tag{22}$$

where

$$\Delta = \sqrt{B^2 + \frac{\epsilon_m^2}{\kappa_0\kappa_m}}. \tag{23}$$

As is seen from (22) the diffraction amplitude $\phi_m(x)$ is proportional to ϵ_m, which is a small parameter in soft x-rays due to dielectric constant $(1+\epsilon)$ being close to 1. However, the diffraction amplitude can be large (of the order of 1) if resonance condition $\mid B \mid \leq \mid \frac{\epsilon_m}{\sqrt{\kappa_0\kappa_m}} \mid$ takes place. In this case $\mid \Delta \mid \ll 1$ and the amplitude ϕ_m as is seen from (22) is actually a slowly varying function of x and assumptions adopted for derivation of (20) from (18) are true. Therefore using (19) and (22) we obtain, for the intensity of the diffracted wave,

$$\mid f_m \mid^2 = \left|\frac{\epsilon_m}{\kappa_m\Delta}\right|^2 \exp[-kx\,\mathrm{Im}(\kappa_0 + \kappa_m)] \times \left[\sinh^2\left(\frac{kx}{2}\mathrm{Im}\Delta\right) + \sin^2\left(\frac{kx}{2}\mathrm{Re}\Delta\right)\right], \tag{24}$$

where κ_m, ϵ_m, and Δ are given by (13), (14), (21), and (23).

With the help of this expression the efficiency of an arbitrary zone plate with slanted zones can be calculated and optimized to provide maximum throughput. This means that optimal materials, their spacing ratio inside the zones, the zone plate thickness, and zone inclination can be determined.

IV. OPTIMIZATION OF ZONE PLATE PARAMETERS

For an example let us consider diffraction efficiency of a zone plate which images the source with 1:1 magnification, which means that $z = z'$ (see Fig. 1). Then using (7) and (13) we obtain $\theta_m = \theta$ and $\kappa_m = \kappa_0 = \sqrt{\cos^2\theta + \epsilon_0}$.

First of all, to provide the resonance effect we define the slanting angle α. As is seen from (24) and (23) the proper choice is $B = 0$, which according to (21) means $\alpha = 0$, i.e., the zones should be horizontal. In this case $\mid \Delta \mid^2 = \mid \frac{\epsilon_m}{\kappa_m} \mid^2$ and the small parameter $\mid \epsilon_m \mid$ in diffraction amplitude (24) is compensated:

$$\mid f_m(\theta) \mid^2 = \exp(-2kx\,\mathrm{Im}\,\kappa_0)\left\{\sinh^2\left[\frac{kx}{2}\mathrm{Im}\left(\frac{\epsilon_m}{\kappa_0}\right)\right] + \sin^2\left[\frac{kx}{2}\mathrm{Re}\left(\frac{\epsilon_m}{\kappa_0}\right)\right]\right\}. \tag{25}$$

Substituting this into (11) and keeping only main terms in expansion of κ_0 and $\frac{1}{\kappa_0}$ in the powers of ϵ_0 we obtain the total diffraction efficiency of the zone plate with horizontal zones and 1:1 magnification:

$$E_{\mathrm{TDE}} = C^{-1}\int_{\theta_1}^{\theta_2} \sin\theta d\theta \exp\left(-\frac{kx}{\cos\theta}[\mathrm{Im}\epsilon_2 + \beta\,\mathrm{Im}(\epsilon_1 - \epsilon_2)]\right) \times\left[\sinh^2\left(\frac{kx}{2\cos\theta}\frac{\sin\pi\beta m}{\pi m}\mathrm{Im}(\epsilon_1 - \epsilon_2)\right) + \sin^2\left(\frac{kx}{2\cos\theta}\frac{\sin\pi\beta m}{\pi m}\mathrm{Re}(\epsilon_1 - \epsilon_2)\right)\right]. \tag{26}$$

Aperture angles θ_1 and θ_2 which according to (11) define the geometrical factor C can be expressed in terms of radius of obscured zone r_0, focal length z, and the total number of zones N with the help of (3) and (6):

$$\cos\theta_1 = \frac{z}{\sqrt{z^2 + r_0^2}}, \quad \cos\theta_2 = \frac{\cos\theta_1}{1 + \frac{\lambda N}{2z}\cos\theta_1}. \tag{27}$$

Two parameters can yet be varied in (26) in order to optimize TDE for given wavelength $\lambda = \frac{2\pi}{k}$, zone plate materials ϵ_1 and ϵ_2, and aperture angles θ_1 and θ_2. They are the spacing ratio β and zone plate thickness x.

A. The choice of the spacing ratio β

To find the optimal value of the spacing ratio β it is convenient to rewrite (26) in the form

$$E_{\rm TDE} = C^{-1}Q\int_{\frac{Q}{\cos\theta_1}}^{\frac{Q}{\cos\theta_2}} \exp\left(-2t\frac{\pi m(g+\beta)}{\sin\pi\beta m}\right) \times[\sinh^2 t+\sin^2(ft)]\frac{dt}{t^2}\,, \qquad (28)$$

where

$$Q = \frac{kx}{2}\frac{\sin\pi\beta m}{\pi m}{\rm Im}(\epsilon_1-\epsilon_2), \qquad f=\frac{{\rm Re}(\epsilon_1-\epsilon_2)}{{\rm Im}(\epsilon_1-\epsilon_2)}, \qquad (29)$$

$$g=\frac{{\rm Im}\epsilon_2}{{\rm Im}(\epsilon_1-\epsilon_2)}.$$

Parameter Q is proportional to zone plate thickness x, and parameters f and g are exactly the same as those which appear in the theory of multilayer mirrors [10,13].

As is seen from (27) the maximum value of TDE is reached when $\frac{g+\beta}{\sin\pi\beta m}$ is minimal, that is,

$$\tan(\pi\beta m)=\pi m(\beta+g). \qquad (30)$$

Again it is the same condition which is true for multilayers. For optimal spacing ratio β which satisfies (30) we have

$$E_{\rm TDE} = C^{-1}Q\int_{\frac{Q}{\cos\theta_1}}^{\frac{Q}{\cos\theta_2}} \exp\left(-\frac{2t}{\cos\pi\beta m}\right) \times[\sinh^2 t+\sin^2(ft)]\frac{dt}{t^2}\,. \qquad (31)$$

So formula (31) gives maximum diffraction efficiency that can be obtained for the zone plate fabricated of materials 1 and 2 and having aperture angles θ_1 and θ_2. The thickness x can also be chosen so as to produce the highest possible diffraction efficiency. This will be considered below.

B. Phase zone plate

The diffraction efficiency of the phase zone plate is obtained by taking in formula (26) ${\rm Im}\epsilon_1={\rm Im}\epsilon_2=0$:

$$E_{\rm TDE} = C^{-1}P\int_{\frac{P}{\cos\theta_1}}^{\frac{P}{\cos\theta_2}} \sin^2 y\frac{dy}{y^2},$$

$$P=\frac{kx}{2}\frac{\sin\pi\beta m}{\pi m}{\rm Re}(\epsilon_1-\epsilon_2). \qquad (32)$$

The diffraction efficiency (32) of phase zone plate which has 2π solid angle ($\theta_1=0,\theta_2=\frac{\pi}{2}$) is reduced to

$$E_{\rm TDE}=P\int_P^\infty\frac{\sin^2 y}{y^2}dy, \qquad (33)$$

and is shown in Fig 4. The maximum efficiency of the thick phase zone plate is reached at $P=0.95$ and equals to 67% for any diffraction order. This value is 1.6 times larger than that known from Kirchhoff-Fresnel theory.

C. Pattern zone plates

These zone plates consist of alternating circular layers of material and vacuum. They are fabricated by lithography or contamination technique including beam drawing and etching [5]. To describe them we are to take ${\rm Im}\epsilon_2\to 0$ and ${\rm Re}\epsilon_2\to 0$ in the formulas obtained above.

The maximum TDE as it is seen from (30) and (31) is obtained for spacing ratio $\beta\to 0$. For a zone plate with 2π aperture this gives

$$E_{\rm TDE}=R\int_R^\infty \exp(-2t)[\sinh^2 t+\sin^2(ft)]\frac{dt}{t^2}, \qquad (34)$$

where

$$R=\frac{kx}{2}\frac{\sin\pi\beta m}{\pi m}{\rm Im}\epsilon_1, \qquad f=\frac{{\rm Re}\epsilon_1}{{\rm Im}\epsilon_1}.$$

The lower limit of (34) occurs for absolutely opaque material ${\rm Re}\epsilon_1 \ll Im\epsilon_1$:

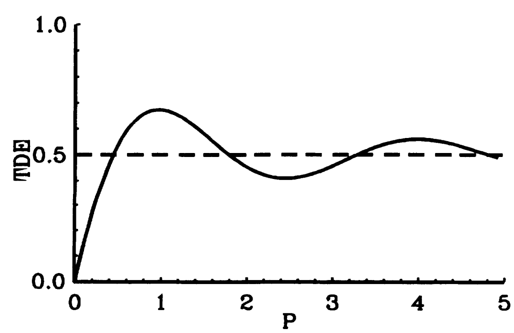

FIG. 4. Efficiency of phase zone plate as a function of thickness [see formula (32)].

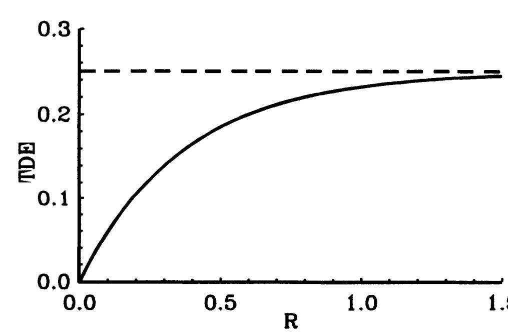

FIG. 5. Efficiency of a zone plate with opaque zones as a function of thickness [see formulas (34), (35)].

$$E_{\mathrm{TDE}} = R \int_R^\infty \exp(-2t) \sinh^2 t \frac{dt}{t^2}. \qquad (35)$$

Maximum diffraction efficiency (35) of a thick pattern zone plate made of absolutely opaque material as shown in Fig. 5 equals 25%. Compare with 10% as it follows from Kirchhoff-Fresnel theory for a thin amplitude zone plate.

V. APPLICATION TO REAL MATERIALS

Let us calculate the maximum total diffraction efficiency which can be achieved with materials usually used now for zone plate and multilayer fabrication.

A. Pattern zone plate

For example, we consider a 2π-aperture zone plate made of Ni which is often used in the range 10 Å< λ < 50 Å. In Fig. 6 the dependence of efficiency upon the thickness is shown for $\lambda = 30$ Å and various spacing ratio β calculated according to (28) and (29). As was mentioned in Sec. IV C and seen from Fig. 6 the efficiency increases if spacing ratio $\beta \to 0$. However, the practical value of β is limited by the total number of zones in the zone plate. Curve (a) in Fig. 7 shows maximum efficiency of the zone plate in dependence on the wavelength λ [see formula (34)]. As is seen, 40% efficiency is possible and $\beta = 0.1$ provides the efficiency close to the maximum value.

B. "Sputter-sliced" zone plate

The zone plates consisting of alternating annular layers of two materials are fabricated by sputtering onto a thin metallic wire used as a substrate and subsequent cutting and etching to obtain required thickness [5]. The promising advantages of such a technology are (a) to advance to a very small width of the outermost zone to obtain high spatial resolution, and (b) to produce zone plates with a high aspect ratio (thick zone plates), which is necessary to increase the zone plate efficiency TDE.

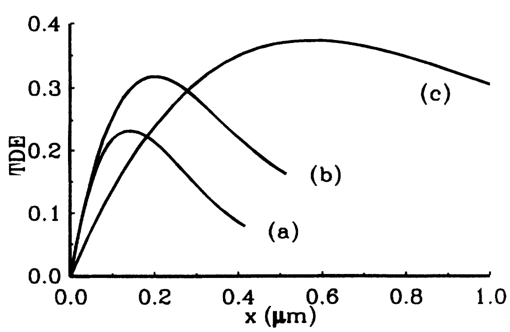

FIG. 6. Efficiency of Ni zone plate at $\lambda = 30$ Å as a function of thickness for various spacing ratios: (a) $\beta = 0.5$, (b) $\beta = 0.3$, and (c) $\beta = 0.1$ [see formulas (28), (29)].

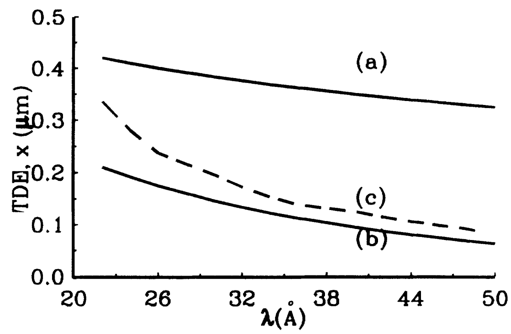

FIG. 7. Curve (a): maximum efficiency of Ni-vacuum zone plate as a function of wavelength λ. Curve (b): the same for Ni-Be zone plate. Curve (c): optimal thickness of Ni-Be zone plate as a function of wavelength λ.

Let us consider a "sputter-sliced" zone plate made of two materials, Ni and Be, which are recognized as good pairs of materials for multilayer mirror fabrication for wavelength range λ < 50 Å. The results of calculations according to (31) are shown in Fig. 7, where the wavelength dependencies of maximum efficiency [curve (b)] and optimal thickness [curve (c)] are presented. The considerable decrease in the efficiency in comparison with a Ni-vacuum zone plate is evidently due to the absorption in Be.

VI. SUMMARY

An analytical theory of thick zone plate efficiency has been developed. This leads to explicit formulas for diffraction efficiency, which enable one to choose materials, their spacing ratio, and the zone plate thickness. The following ideas and assumptions were essential in our consideration: (a) the zone plate is treated as a grating with slowly varying period, (b) locally, in small areas the zone plate has the structure of a sliced multilayer, and (c) the full wave equation can be replaced (truncated) by the system of two coupled wave equations, which has exact analytical solution. Assumptions (a)–(c) are valid for a large class of zone plates, including those produced by existing technology and used in x-ray, neutron, and atom optics. The same approach can be employed for design of zone plates having magnification $M \neq 1$ intended for application in imaging and scanning microscopy and also for zone plates with variable thickness, zone profiles (e.g., sawtooth), or zone transmission functions to obtain the enhanced diffraction efficiency.

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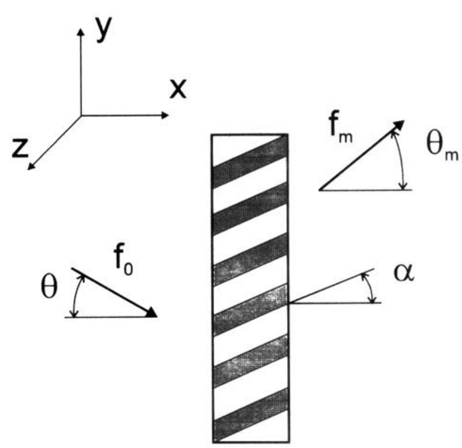

FIG. 2. A local grating of the zone plate with slanted zones.